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The Effect of Force Ratio Multiplier on A Control System for Surfing Problem Simulation

Reza Rendian Septiawan[1*], Agus Virgono[2], Umar Ali Ahmad[3], Prayitno Abadi[4], Mas'ud Adhi Saputra[5]

[1,2,3,4,5]Computer Engineering Study Program, Faculty of Electrical Engineering, Telkom University, Bandung, 40257, Indonesia
[2]Atmospheric Science Center, BRIN-LAPAN, Bandung, 40173
[3]The Office of Engineering Materials and Products, Ministry of Industry, Bandung, 40135

Abstract

A surfing problem is a control problem of the surfing board to maintain its position on top of an ocean wave as long as possible. There are some physical and mathematical problems that have not yet been solved regarding a surfing problem. One of them is on translating a target inclination problem from an ordinary differential equation (ODE) control system to the inclination of a surfing board via the distribution of a surfer's weight. To move the surfing board swiftly, a correct value of the multiplier, which is notated by σ, is needed on the weight distribution system. In this work, an investigation has been done on the effect of the multiplier in an attempt to help moves the surfing board and fulfils the target inclination angle needed in a simulation by using a smoothed particle hydrodynamics (SPH) method. The result from this work shows that the best value for the multiplier is $\sigma = 10$ that gives the smallest average positional error of 0.0412 meter for some variations of a given target position. This work contributes on a better mathematical model for a surfing problem.

Keywords: *Fluid simulation; hydrodynamics; ODE; SPH; surfing problem*

1. Introduction

A surfing problem is a physics problem emerges on a surfing sport when the surfer tries to balance their surfing board on top of the ocean wave as long as possible. The surfing board is balanced by shifting the location of surfer's feet, i.e., shifting their body weight distribution on top of the board to counterbalance external forces that come from the ocean wave. Mathematical modelling can be done to understand the motion of the surfing board.

A surfing problem is a relatively new research problem. Previously, research on a mathematical model of a surfing problem is already done in [1] by simulating an ordinary differential equation (ODE)-based control system which manipulates the inclination angle of the surfing board depends on a position, velocity, and observed inclination angle of the surfing board. But some problems still occur; one of them is about the swiftness of the control system. The control system is still not fast enough in controlling the surfing board on fulfilling its target inclination angle which is the

*Corresponding author. *E-mail address: zaseptiawan@telkomuniversity.ac.id*
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output of the control system. A solution that can be done to overcome that problem is by giving a correct multiplier on the surfer's weight distribution system which depends on the input data so that the surfing board can tilt and follow the target inclination angle. In this research, the effect of the multiplier on the surfer's weight distribution system is observed on maneuvering the surfing board. Later, the capability of the control system is validated via fluid simulations of the whole system which make use of the smoothed particle hydrodynamics (SPH) method.

Particle simulation methods are frequently used to solve various physics problems which need material discretization with information that are stored and moved together with particles (Lagrangian frame), e.g., calculation of porosity for material deposition process [2]. Smoothed particle hydrodynamics (SPH) is one the most popular methods commonly used on solving various problems related with fluid. In the beginning, SPH is introduced to solve astrophysical problems [3], [4]. After that, SPH is widely used on solving fluid-related physical problems via computational fluid dynamics (CFD). Some examples of problem solved by an SPH method are ice melting problem [5], fluid natural convection problem [6], fluid with high viscosity [7], and ocean wave simulation [8], [9].

A fluid-solid interaction problem Interactions between fluid and solid is a compelling problem in a computational fluid dynamics (CFD) field to be solved by using an SPH method. Some research regarding this problem have been done previously on [10], [11]. One method that can be used to represent the interaction between fluid particle and solid in a particle method is by discretizing the solid into a set of particles with similar properties with fluid particles and can interact with fluid particles with the same interaction as fluid-fluid interaction. On some cases, the fluid-solid interaction needs to be tweaked to suit the problem. The common problem with fluid-solid interaction is penetration of fluid particles into solid which can be prevented by implementing an external repulsive force between fluid particles and solid particles. There are several repulsive force schemes that can be applied to such a problem, one of them is a Monaghan boundary force (MBF) scheme which is introduced in [12]. But since MBF scheme is based on the fastest relative speed between fluid particles and solid particles, the magnitude of MBF repulsive force is occasionally too big for relatively slow-moving particles. A better method is introduced in [13] which includes particles on domain boundaries in a particle density-updating step which also solves a particles-deficiency problem near boundaries.

On a surfing problem simulation, a fluid-solid interaction is intensively occurred between an ocean wave and a surfing board. In this research, a fluid-solid interaction is done by using a pure hydrodynamic force. A solid body is going through a discretization process and is turned into a set of particles. Solid particles interact with fluid particles utilizing the exact same interaction between fluid-fluid particles. The fluid particles penetration problem is solved by fixing the density of solid particles.

The goal of this research is to improve an ODE model for a control system in a surfing problem, especially in a swiftness of a surfing board's movement. To get a better action, the effect of a weight distribution's multiplier on a control system is studied here so it can mimic the maneuver of a surfing board in a real world. In short, to find the optimal value of the multiplier, some simulations with different values of a multiplier are executed and their result is analyzed. The more detail explanation about the method used here is discussed in the upcoming parts.

2. Methods

A Smoothed particle hydrodynamics

Smoothed particle hydrodynamics (SPH) is a method that mathematically approximating a field function $f(x)$ by convoluting those field function with a smooth-enough mollifier function $\Psi(x)$ which is also known as a kernel function. The approximation can be written as Eq. 1,

$$f(x) \approx \int_{\mathbf{R}^n} f(y)\Psi_h(x-y)dy, \tag{1}$$

where $\Psi_h(x) := \frac{1}{h}\Psi(\frac{x}{h})$ is a kernel function which distribution is determined by a parameter h. When $h \to 0$, a function $\Psi_h(x)$ will be more concentrated on an origin point compared with the original function $\Psi(x)$ [14]. The approximation for a derivative of $f(x)$ is written as Eq. 2,

$$\partial^\alpha f(x) \approx \int_{\mathbf{R}^n} f(y)\partial^\alpha \Psi_h(x-y)dy. \tag{2}$$

A kernel function used in this research is a compact-supported cubic kernel function which is second-order differentiable which can be written as Eq. 3,

$$\Psi(x) = \frac{1}{4\pi}\begin{cases} (2-|x|)^3 - 4(1-|x|)^3, & 0 \leq |x| < 1 \\ (2-|x|)^3, & 1 \leq |x| \leq 2 \\ 0, & 2 \leq |x| \end{cases} \tag{3}$$

for a 3-dimension case.

To solve the problem numerically with an SPH method, Equation (1) and (2) must be discretized. A discretized form of a field function $f(x)$ is written in Eq. 4,

$$f(x) \approx \sum_{i=1}^{N} f(r_i)\Psi_h(x-r_i)V(E_i). \tag{4}$$

And a discretized form for a derivative of a field function f(x) is written in Eq. 5,

$$\partial^\alpha f(x) \approx \sum_{i=1}^{N} f(r_i)\partial^\alpha \Psi_h(x-r_i)V(E_i), \tag{5}$$

where $V(E_i)$ is a volume of a set E_i around a point r_i The most used approximation for $V(E_i)$ is written in Eq. 6,

$$V(E_i) = \frac{m_i}{\rho_i}, \tag{6}$$

where m_i dan ρ_i are a mass and density in a point r_i, respectively. Equation (4) and (5) will be used to approximate governing equations that control the movement of all particles in the system.

B **Fluid dynamics**

SPH method models a system in the Lagrangian specification. Governing equations used here for modeling fluid dynamics in the Lagrangian specification are [15]:

(a) Mass conservation, which can be written as Eq.7,

$$\frac{D_\rho}{D_t} = -\rho \operatorname{div}(u) \tag{7}$$

(b) Momentum conservation, which can be written as Eq. 8

$$\frac{D_u}{D_t} = -\frac{1}{\rho}\nabla_p + b \tag{8}$$

Equation 7 and 8 are parts of a set of Navier-Stokes equations. An energy conservation equation is not used here since there is no significant energy transfer. A momentum equation used here is a momentum equation for an inviscid fluid. Viscosity of the fluid in our case is modeled directly into the SPH method by using a numerical viscosity. $\frac{D}{D_t}$ operator is a substantial derivative operator defined as Eq. 9,

$$\frac{D_f}{D_t} = \frac{\partial f}{\partial t} + u.\nabla f, \tag{9}$$

for each field function $f(x,t)$ and field velocity u. A substantial derivative operator shows a change of value of field function $f(x,t)$ with respect to time as the entity that brings those aforementioned field function moves in a field velocity u.

One of the most common SPH approximation for a mass conservation equation is the one in its anti-symmetric form which is used in [16]. The approximation can be written as Eq. 10,

$$\frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j}(u_i - u_j).\nabla\Psi_h(r_i - r_j). \tag{10}$$

with m_j, ρ_i, and u_i are mass, density, and velocity in a point r_j and time t, respectively. An SPH approximation for a momentum conservation in its anti-symmetric form which is also used in [16] can be written as Eq. 11,

$$\frac{du_i}{dt} = -\sum_{j=1}^{N}\left(m_j\left(\frac{{p_i}^2}{\rho_i} + \frac{p_j}{\rho_j^2}\right)\nabla\Psi_h(r_i - r_j)\right) + b_i, \tag{11}$$

with p_i is a pressure in a point r_i at time t. SPH approximations for mass and momentum conservation which are shown in (10) and (11) are used in this research.

Pressure in a point r_i shown in Eq. 13 is calculated by using a Tait's relation equation which shows the relation between a fluid density and a fluid pressure. Tait's relation equation for an SPH method can be written as Eq. 12 [17], [18], [19],

$$p_i = \frac{c^2\rho_0}{\gamma}\left(\left\{\frac{\rho_i}{\rho_0}\right)^{\gamma} - 1\right), \tag{12}$$

where ρ_0 is a reference density, c is a speed of sound in a fluid, and $\gamma = 7$ is a constant used for a water-like fluid.

C Rigid body dynamics

A rigid body which has a total mass M which is time-independent can be defined as Eq. 13,

$$M = \int_{\mathbf{R}} \rho(x)dx, \tag{13}$$

where ρ is a density and $\mathbf{R}$ is a configuration of a rigid body in a principal frame. If a rigid body moves with a velocity $U(t) = \frac{d}{dt}X(t)$ with $X(t)$ is a position of a center of mass of a rigid body, a linear momentum of those rigid body can be written as Eq. 14,

$$G(t) = MU(t), \tag{14}$$

Let us define $F^i(t) \equiv F(x^i, t)$ as a force works on a point x^i located on a rigid body at a time t and $F(t) = \sum_i F^i(t)$ is a total external force works on a rigid body, a relation between a total force works on a rigid body and a rate of change of a linear momentum can be written as Eq. 15,

$$F(t) = \frac{d}{dt}(MU(t)) = M\frac{d}{dt}U(t) = MA(t) \tag{15}$$

where $A(t)$ is a linear acceleration of a rigid body at a time t.
A rotational dynamic of a rigid body can be written in a Euler equation for a rigid body as Eq. 16 [20],

$$\hat{K}(t) = \hat{\omega}(t) \times J\hat{\omega}(t) + J\bar{\hat{\omega}}(t), \tag{16}$$

where $\hat{K}(t) = \mathbf{R}^T(t)K(t), \hat{\omega} = R^T(t)\omega(t)$, and $\bar{\hat{\omega}}(t) = \frac{d}{dt}\hat{\omega}(t).\mathbf{R}(t)$ is a rotational matrix, $\omega(t)$ is an angular velocity vector, and a total moment of force (torque) $K(t) =$

$\frac{d}{dt}(\mathbf{R}(t)J\mathbf{R}^T(t)\omega(t))$is a rate of change of angular momentum through time. J is a principal moment of inertia tensor of a rigid body. The diagonal components of J are written in Eq. 17,

$$\begin{cases} J_1 & := & J_{11} = \int\limits_{\mathbf{R}} \rho(x)(x_2^2 + x_3^2)dx, \\ J_2 & := & J_{22} = \int\limits_{\mathbf{R}} \rho(x)(x_1^2 + x_3^2)dx, \\ J_3 & := & J_{33} = \int\limits_{\mathbf{R}} \rho(x)(x_1^2 + x_2^2)dx, \end{cases} \tag{17}$$

In this research, the interaction between a surfing board and an ocean wave is done via the fluid-solid particles interaction. A surfing board which acts as a rigid body is discretized into a set of particles and interact with fluid particles. The difference between solid particles and fluid particles is in the density. Solid particles density is never updated and always be the same with the solid reference density, which results in zero interactions between solid particles.

Same as fluid particles, solid particles also bring information about physical quantities which are used to calculate the dynamic of a rigid body. In time t, the position vector of each particle i of a rigid body can be written as $r_i(t) = (r_{i,1}(t), r_{i,2}(t), r_{i,3}(t))$. The position vector $r_i(t)$ is used to calculate inertia tensor J which is used on many calculations for rigid body dynamics. Diagonal components of J in a discretized form can be written as Eq. 18,

$$\begin{cases} J_{11} & = & \rho_b \sum_{i=1}^{N_b}(x_{i,2}^2 + x_{i,3}^2)dx, \\ J_{22} & = & \rho_b \sum_{i=1}^{N_b}(x_{i,1}^2 + x_{i,3}^2)dx, \\ J_{33} & = & \rho_b \sum_{i=1}^{N_b}(x_{i12}^2 + x_{i,2}^2)dx, \end{cases} \tag{18}$$

and non-diagonal components can be written as Eq. 19,

$$\begin{cases} J_{12} & = & J_{21} = -\rho_b \sum_{i=1}^{N_b} r_{i,1}r_{i,2}dx, \\ J_{13} & = & J_{31} = -\rho_b \sum_{i=1}^{N_b} r_{i,1}r_{i,3}dx, \\ J_{23} & = & J_{32} = -\rho_b \sum_{i=1}^{N_b} r_{i,2}r_{i,3}dx. \end{cases} \tag{19}$$

The interaction between a rigid body and fluid particles is calculated by using a pure hydrodynamics force. Total force works on a rigid body particle in a position r_i can be written as Eq. 20,

$$f_i = -m_i c_V \left(\left(\sum_{j=1}^{N} m_j \frac{p_j}{p_j^2} \nabla \Psi_h(r_i - r_j) \right) + b_i \right) \tag{20}$$

where $c_V = \frac{h^3}{V(E_i)}$ is a multiplier used to handle a volume discrepancy issue between a volume on an SPH calculation ($V(E_i)$) with a volume of a cube with a length h. After that, a movement of a rigid body is done by calculating total moment of force K(t) and updating the position of all rigid body particles by following the scheme in [1].

D Mathematical modelling of a surfing problem

The goal of the surfing is to maintain the position of the surfer to be on top of the ocean wave as long as possible by controlling the inclination angle of the surfing board. The inclination angle is controlled by using an ODE-based control model which in general can be written as Eq. 21,

$$\dot{\theta}(t) = a(z(t) - \tilde{Z}) + b(V(t) - \tilde{V})_c(\theta(t) - \tilde{\theta}) \tag{21}$$

where $Z(t)$ and $V(t)$ are third components of a position vector and a linear velocity vector, respectively, in a time t. $\theta(t)$ is an inclination angle of a surfing board in a time $t.\tilde{Z}, \tilde{V}$, and

$\tilde{\theta}$ are target values for a position, a linear velocity, and an inclination angle, respectively. a, b, and c are constant multipliers related with a position, a linear velocity, and an inclination angle, respectively. $\dot{\theta}$ is a derivative of an inclination angle with respect to time.

In a stable position, when a surfing board is already in a target position, a surfing board should not move anymore, or in another word, its linear velocity should be zero, $\tilde{V} = 0$. A target position, $\tilde{Z}$, is a given parameter. $\tilde{\theta}$ is a target inclination angle needed to stabilize a surfing board in its target condition. The frame of the system in simulations is illustrated in Figure 1.

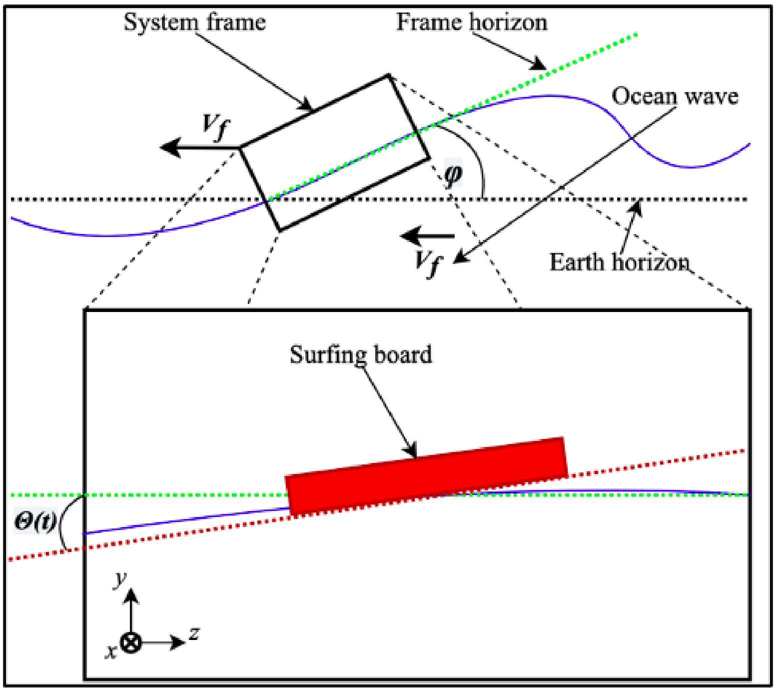

Figure 1. Illustration of the frame of the system.

A linear velocity is defined as a derivative of position with respect to time, $\dot{Z}(t) = V(t)$. Assuming the system is close to a stationary condition, an acceleration of the system can be modeled as a linear function of a position, a linear velocity, and an inclination angle of a surfing board. Denoting $\xi(t) = (Z(t), V(t), \theta(t))$ as an unknown parameter in the system, a simplified ODE model of the system can be written as Eq. 22,

$$\left\{ \begin{array}{rcl} \dot{Z}(t) & = & V(t) \\ \dot{V}(t) & = & -\mu\theta(t) - \mu_v V(t) - \mu_z Z(t) - \mu_0, \\ \dot{\theta}(t) & = & a(Z(t) - \bar{Z}) + bV(t) + c(\theta(t) - \bar{\theta}), \end{array} \right. \tag{22}$$

where μ, μ_v, μ_z, dan μ_0 are constants related to variables that affect linear velocity. By simplifying $\mu_v = \mu_z 0$ and analyzing the stability of the system mathematically, constant multipliers a, b, and c can be set to be $a = \frac{60}{\mu}, b = \frac{46}{\mu}$, and $c = -12$ [1].

A target inclination angle as an output of the ODE control system is an inclination angle aimed by a surfer. To satisfy a target inclination angle, a surfer needs to correctly distribute their body weight force on a surfing board. Those body weight force distribution control system is a main focus of this research. Body weight force distribution system can be simply modeled by two forces given on two contact points on a surfing board located at both of its edges. Total magnitude of both forces must be the same with the weight of a surfer. Locations of both contact points can be seen on Figure 2. Total magnitude of both forces must be the same with the weight force of a surfer. The ratio of both forces can be written

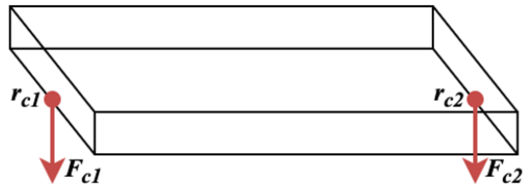

Figure 2. Locations of both contact forces on a body weight force distribution system.

as Eq. 23 and 24,

$$F_{c1} = T(t)W, \tag{23}$$

$$F_{c2} = (1 - T(t))W, \tag{24}$$

where $T(t)$ is a function that determines the ratio of magnitude of both forces and W is a weight forces of a surfer. Function $T(t)$ can be defined as Eq. 25,

$$T(t) = 0.5 - \sigma(\hat{\theta}(t) - \min(\theta(t), \theta_m)), \tag{25}$$

where $\theta(t)$ is a target inclination angle from the ODE control system on Eq. 20, θ_m is a maximum allowed inclination angle, and $\hat{\theta}(t)$ is an observed inclination angle at a time t. σ is a parameter that controls the rate of change of weight forces ratio with respect to the difference between an observed inclination angle with a target inclination angle. A clipped function is used to prevent a surfing board becomes too inclined that can cause a surfing board sunk into an ocean wave. An inclination angle is chosen to be a negative value when a surfing board is inclined toward a sky (upward direction).

E Simulation algorithm

In general, for each timesteps in a simulation consists of a calculation of interparticle interactions with an SPH method continued with an ODE solving to get a target inclination angle. After that, the target inclination angle is forwarded to a surfing board through a body weight force distribution function. Next, rotational dynamics of a rigid body is calculated and ended with a position updating process for all particles in the system. Those cycle is done up to a given maximum simulation time. The algorithm of the simulation can be seen on Figure 3.

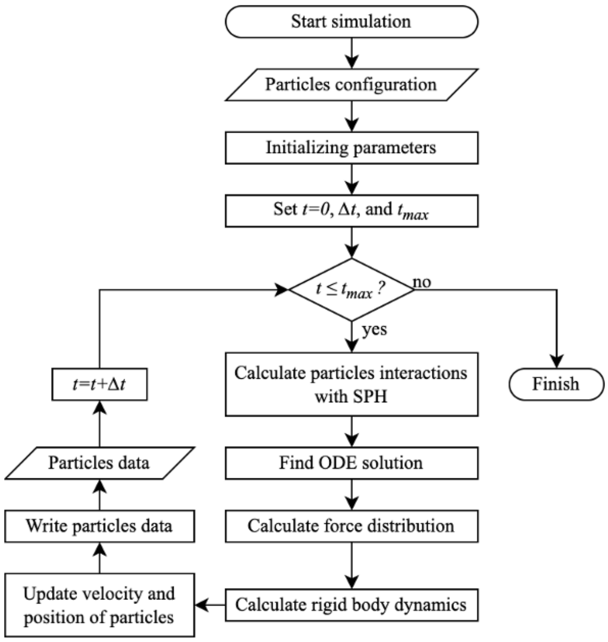

Figure 3. The algorithm of the simulation with an SPH method.

3. Result and Discussion

An initial configuration of the system used in this research can be seen on Figure 4(a). Before a simulation started, a relaxation process is done to minimize the energy of the system so that the simulation can be started on a more stable condition. An initial condition after a relaxation process can be seen on Figure 4(b).

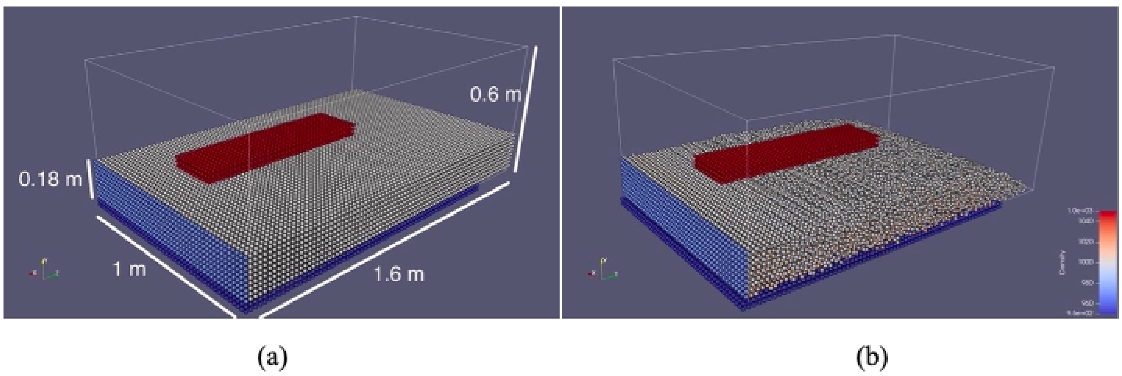

Figure 4. An initial configuration of the system: (a) before relaxation, and (b) after relaxation.

Simulation is run with reference density of the water is equal to $\rho_0 = 1000$ kg.m/s and for a solid body is $\rho_b = 100$ kg.m/s. Fluid is initialized with an initial velocity toward $+z$-axis with magnitude $V_f = 2.5$ m/s. A kernel function used is a cubic-spline kernel shown in Eq. 3 with a kernel parameter is set to $h = 0.02$ m. All particles are initialized in grids with the length of each grid is set to equal with h. Mass of all particles is set to $m_i = 0.008$ kg. The size of a timestep is $\tau = 0.0005$ s with a speed of sound in a fluid is set to $c = 20$ m/s. We assume that the fluid is inviscid.

The system also has a free boundary condition which is implemented into an SPH method by using a ghost point technique. Fluid particles that leave a domain up to h will be marked as ghost points. Ghost points still interact with other particles as usual, but their density is fixed to be the same with reference density and they will not undergo any changes in their velocity. If a ghost point enters the domain again, its status will be returned into a usual fluid particle. In contrast, any ghost points that are going further than h from the domain will be erased from the system.

To find the best value of σ, here simulations with combinations of various parameters are run to obtain a combination of parameters that gives a smallest error. An error here is an average of differences between a third component of vector position of a surfing board (represented by Z) and a target position (represented by $\tilde{Z}$). Simulations are done with a value of parameter σ in a range of $\sigma \in \{0.5, 1, 2, 5, 10, 20, 50, 100\}$ for each $\tilde{Z} \in \{-0.6, -0.5, -0.4, -0.3, -0.2\}$. The difference is calculated for each timesteps of simulations and averaged out toward a whole simulation time. An average of positional error can be seen on Table 1.

Table 1. An average of positional error of a surfing board for different σ.

$\bar{Z}/\sigma$	0.5	1.0	2.0	5.0	10.0	20.0	50.0	100.0
-0.6	$8.93e^{-02}$	$8.60e^{-02}$	$5.69e^{-02}$	$5.163e^{-02}$	$8.03e^{-02}$	$1.31e^{-01}$	$6.78e^{-01}$	$7.00e^{-01}$
-0.5	$8.01e^{-02}$	$8.90e^{-02}$	$4.59e^{-02}$	$4.59e^{-02}$	$4.49e^{-02}$	$2.84e^{-01}$	$5.78e^{-01}$	$6.00e^{-01}$
-0.4	$1.43e^{-01}$	$1.31e^{-01}$	$4.39e^{-02}$	$4.39e^{-02}$	$3.03e^{-02}$	$1.84e^{-02}$	$4.78e^{-01}$	$5.00e^{-01}$
-0.3	$2.20e^{-01}$	$1.35e^{-01}$	$3.08e^{-02}$	$3.08e^{-02}$	$2.77e^{-02}$	$1.72e^{-01}$	$1.93e^{-02}$	$1.63e^{-02}$
-0.2	$2.79e^{-01}$	$1.42e^{-01}$	$3.95e^{-02}$	$3.95e^{-02}$	$2.52e^{-02}$	$2.71e^{-01}$	$2.61e^{-01}$	$2.60e^{-01}$
Average	$1.62e^{-01}$	$1.13e^{-01}$	$6.76e^{-02}$	$4.32e^{-02}$	$4.17e^{-02}$	$1.26e^{-01}$	$4.03e^{-01}$	$4.15e^{-01}$

The best value of σ can be determined by calculating the average of average positional differences for all values of $\tilde{Z}$ for each value of σ. Those average value can be seen on the last row in Table 1. Based on the data, when $\sigma = 10$ the smallest value of average of average of positional error is obtained with value $4.17e^{02}$. The graph of a surfing board's position vs. time for various combinations of parameters can be seen on Figure 5.

Figure 5 shows a position of a surfing board in a z-axis vs. time for various combination of σ dan $\tilde{Z}$. In general, by using smaller values of σ could prevent overshooting occures on the control system. But smaller values of σ give slow convergences on the system [21]. In our case, σ can be seen as a proportional gain parameter (K_p) on a proportional control system [22] in Eq. 26,

$$u(t) = K_p e(t) \tag{26}$$

where $u(t)$ dan $e(t)$ are an output of a control system and an error from input signal on a control system at time t.

On the other hand, larger values of σ give a large control action to the system which leads to a faster convergence with its own downside: a higher probability of overshoot occurrences. Overshoot phenomenon can be seen easily in Figure 5 starting from $\sigma = 20$. It can be seen clearly from the graph that for $\tilde{Z} = -0.2, \tilde{Z} = -0.4$, and $\tilde{Z} = -0.6$ a surfing board is drifting away out of the system frame by the ocean wave as a result of an overshoot that happened on a control system. As the value of σ is getting higher, an overshoot happens faster. An overshoot is a common thing happens in a proportional control system as the value of a multiplier K_p increases. The comparison between a simulation without a control system and a simulation with a good control system can be seen on Figure 6 and Figure 7.

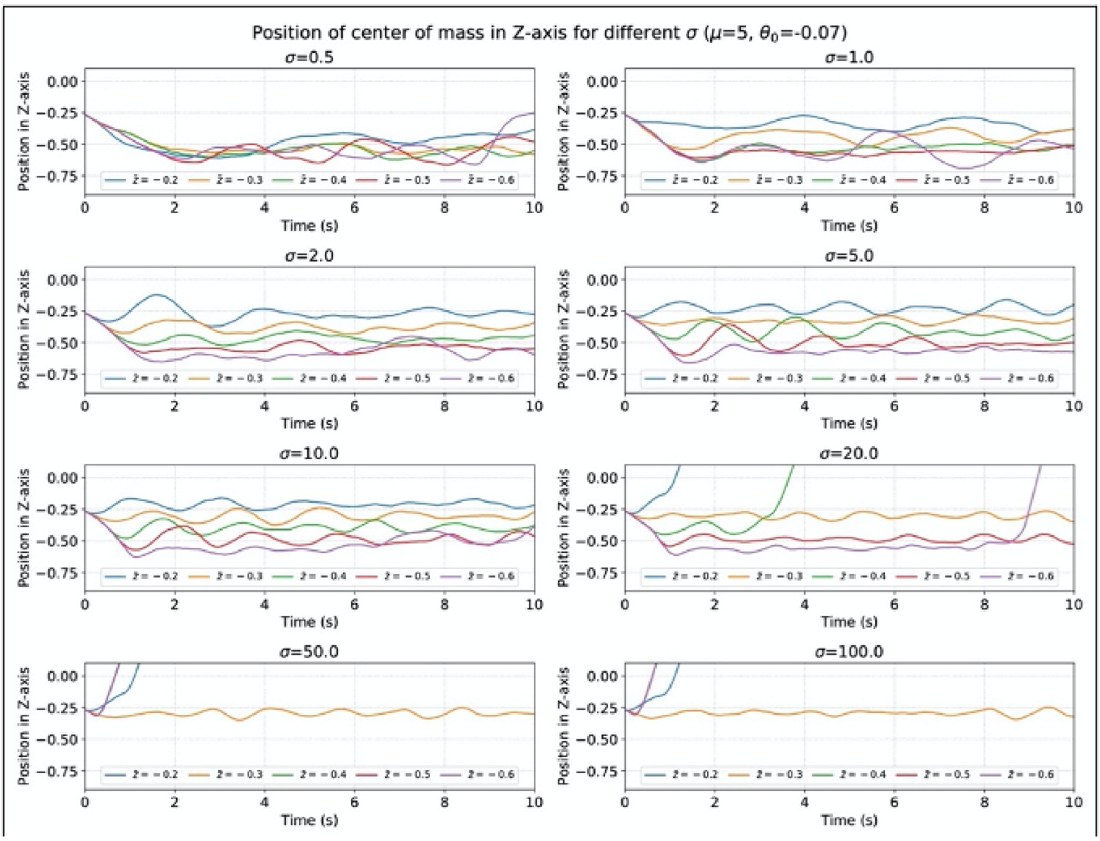

Figure 5. The third component of a surfing board's position vs. time for various value of σ.

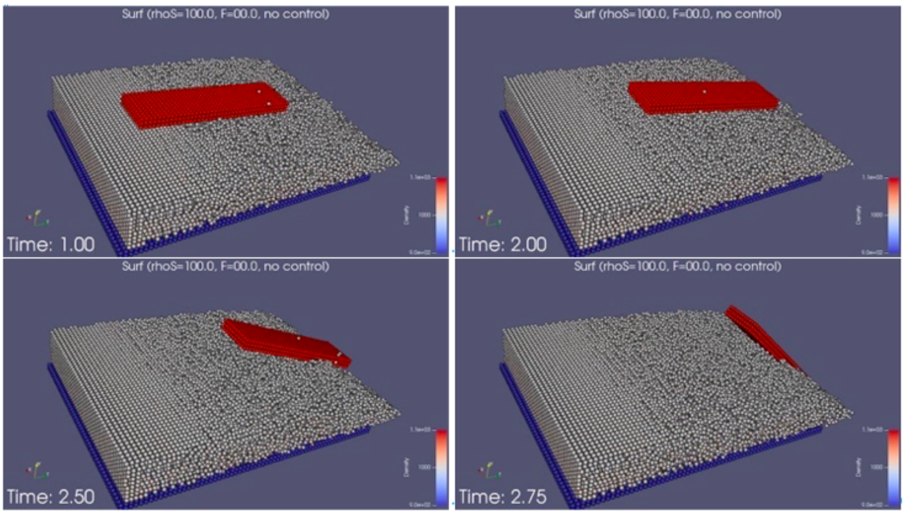

Figure 6. A visualization of a simulation without a control system.

Overshoot problems in our case can be managed by using a more sophisticated control system which can do a more fine-tuned control system. The weight force distribution control system can be modified to implement a proportional-integral-derivative (PID) control system which can give a better result since it utilizes the result from previous timesteps to predict and control the output of a control system in upcoming timesteps. Another control system that can be used is a fuzzy PID control system that can give better control on a non-linear system [23].

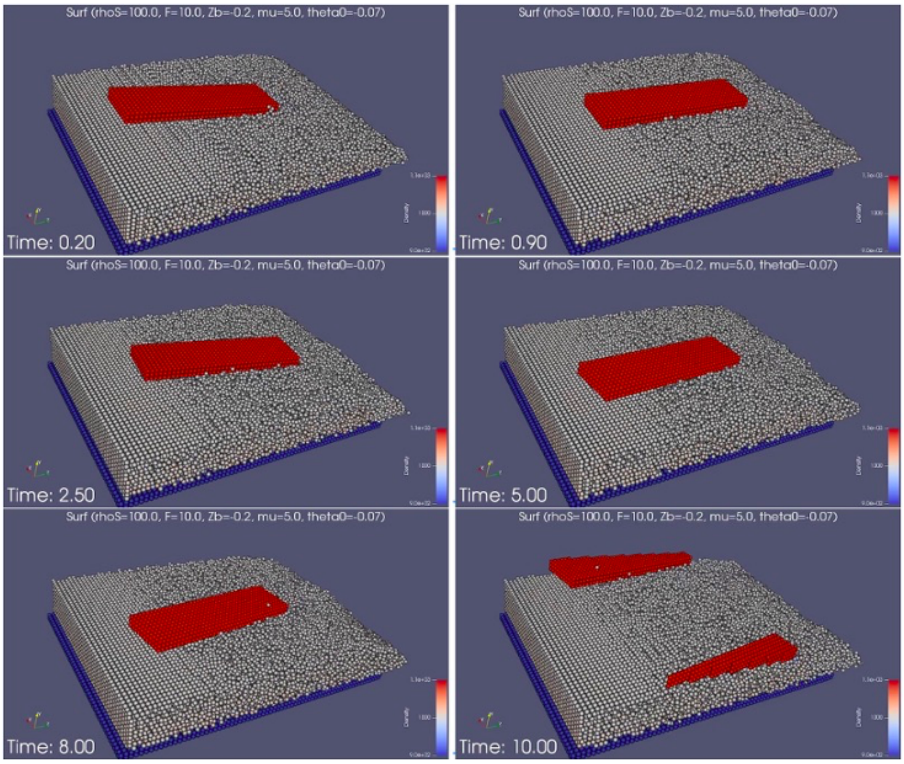

Figure 7. A visualization of a simulation that utilizes a control system with parameters $\tilde{Z} = -0.2, \mu = 5, \tilde{\theta} = -0.07$, dan $\sigma = 10$ on some timesteps.

4. Conclusion

The implementation of an ODE-based control system on a surfing board simulation by using an SPH method is successfully done with good results, proven by its ability on controlling a movement of a surfing board so it can maintain its position to be always on top of the ocean wave with some parameters values. One of many parameters that determine its successfulness is σ which is a multiplier on a weight force distribution control system on a surfing board. On our example case, the best value for σ is $\sigma = 10$ which gives the smallest average of an average of positional error 4.17×10^{-2} for 10-second simulations The occurrences of overshoot problems still observed in simulations with a control system, especially with larger values of σ. That problem can be managed by using a better control system on a weight force distribution control system, such as by implementing a PID control system.

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