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Zillmer Reserves in Dwiguna Life Insurance with the Cox-Ingersoll-Ross (CIR) Interest Rate

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Abstract

Abstract This article discusses Zillmer's reserve of dual-purpose life insurance. Zillmer reserve is one type of modified premium reserve that is calculated using prospective reserves and Zillmer level, which is the difference from gross premium and net premium in the 1st policy year is greater than standard loading. It is necessary to find a way for the loading to be smaller than standard loading. The purpose of this research is to determine the cash value of an annuity using the Cox-Ingersoll-Ross (CIR) interest rate. Furthermore, the method of determining Zillmer's reserves is carried out using the CIR interest rate which will be expressed in the form of a discount factor by estimating two parameters through the variance. As a result, the Zillmer Reserve formula is obtained by using the CIR interest rate, for insured participants who are many years old, with a coverage period of many years, and payments for many years with a certain Zillmer rate value. This research is useful for determining the amount of reserves owned by insurance companies, so that reserves are sufficient in the event of an insurance claim.

Keywords: *Dwiguna Life Insurance; prospective reserves; Zillmer reserves; CIR interest rates*

1. Introduction

Dwiguna life insurance is life insurance whose coverage period is determined, both inside and at the end of the coverage period, both death and survival[2][6][11], the insured still gets the sum assured. In an insurance company, when a person becomes an insurance participant, he should pay premiums to the insurance company [8]. From the premium, several incomes will be obtained from interest during the premium payment period. The income from the premium will later be used to pay for a number of needs from the insurance company. Reserves are the amount of money that is in the insurance company within the coverage period [7]. Reserves that exist in the insurance company will be used in the payment of the sum assured to insurance participants. In determining the reserve, premiums and annuities for life are required which are influenced by the probability of living and the probability of dying. Life opportunities for insurance participants. Based on the

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time of premium calculation, the reserves are divided into two, namely based on the past which is called retrospective premium reserves, and based on the future called prospective premium reserves. Prospective reserves are reserves that are calculated based on the present value of all future expenses minus the present value of total future income for each policyholder. Hasriati [9] in a previous study discussed the determination of the Dwiguna Life Insurance Prospective Reserve. Prospective dual-purpose life insurance premium reserves for someone aged x years where t is the reserve calculation time, m is the premium payment period, and n is the coverage period. At the beginning of the policy year, it usually costs quite a lot to pay for various needs for example to pay fees to premium collectors and others. Therefore, we need a way so that insurance companies do not suffer losses, namely by collecting funds obtained from reserves. In this study, Zillmer Reserves [7] will be used, which is one type of modified premium reserves whose calculations use prospective reserves and the Zillmer level of α. Furthermore, the determination of Zillmer's reserves uses a single premium and annuity cash value. To determine the cash value of an annuity, this paper uses the interest rate [4], where the CIR interest rate has a tendency to return to the long-term average of the interest rate. The discount factor in the CIR interest rate model is influenced by the average and variance of the CIR interest rate.

2. Methods

2.1. Survival function, Probability of Death and Chance of Life

Definition 2.1 Random variable X is said to be a continuous random variable if there is a function f(x), so that its cumulative distribution function is expressed as

$$F(x) = Pr(X \leq x) = \int_{-\infty}^{x} f(u)du.$$

In Actuarial, there is the term survival function which is referred to as a survival function. The survival function is denoted by S(x), expressed as

$$S(x) = Pr(X > x)$$

So based on Definition 2.1, the relationship between the survivals function and the distribution function is obtained as follows [1]:

$$S(x) = 1 - Pr(X \leq x),$$
$$S(x) = 1 - F(x) \tag{1}$$

In the field of actuarial science, the survival function $S(x)$ expresses the probability that a person will live up to x years, where X is a continuous random variable that represents a person's survival time. The survival function of life insurance participants depends on the remaining age-related to time, so it will be viewed as a random variable [12]. Suppose $T(x)$ is a continuous random variable that represents the remaining life time of a person aged x and t represents the time period, then the distribution function of the random variable $T(x)$ can be expressed as

$$F_{T(x)}(t) = Pr[T(x) \leq t], t \geq 0$$

In this case, the function $F_{T(x)}(t)$ represents the probability that a person aged x will die in t years later, then $F_{T(x)}(t)$ is denoted by tq_x. Based on equation (1), the survival function can also be expressed as

$$S_{T(x)}(t) = 1 - F_{t(x)}(t)$$
$$S_{T(x)}(t) = 1 - tq_x$$

The relationship between the probability of living and the probability of dying for an insurance participant aged x years can survive up to t years later based on equation (1) can be expressed as

$$ {}_tP_x = 1 -_t q_x $$

Suppose l_x represents the number of people aged x years and l_{x+t} represents the number of people aged x who are still alive until t years in the future [3], then the probability a person of age x can survive up to t years later can also be expressed as

$$ {}_tP_x = \frac{l_{x+t}}{l_x} $$

In determining the amount of a single premium, the discount factor and the probability of dying are delayed. Suppose an insurance participant aged x years will survive until t years and will die in the next u years, meaning that x dies between the ages of (x+t) and (x+t+u) years. The probability of dying pending is stated as[5]

$$ {}_{t|u}q_x = Pr[t < T(X) \leq t + u] $$

$$ {}_{t|u}q_x =_t P_x -_{t+u} P_x $$

If $u = 1$, then a person who is x years old will live to t years and will die for the next 1 year is

$$ {}_{t|1}q_x =_t P_x -_{t+1} P_x $$

2.2. Life Annuity in Life Insurance

Annuity is a series of payments in a certain amount made every certain time interval as long as the person is still alive [10]. Annuities are divided into two, namely annuities whose payments are made at the beginning of the period called the initial annuity, and annuities whose payments are made at the end of the period which is called ending annuities. In this study, the type of annuity used is a term life annuity whose payment is made at the beginning of the period. In determining the size of the annuity, it is strongly influenced by the interest rate which is denoted by i. In interest there is a function v called the discount factor, namely the present value of the payment of 1 unit of payment made one year later and expressed as

$$ v = \frac{1}{1+i} $$

The cash value of an early life annuity term is denoted by $\ddot{a}_{x:\overline{n|}}$, where n represents the period of time and x represents the age of the insurance participant. Suppose v is the discount factor, represents the probability that the insurance participant aged x years will live until t years later and n is the term of the annuity payment, then the cash value of the early life annuity term for 1 unit of payment is expressed as

$$ \ddot{a}_{x:\overline{n|}} = \sum_{t=0}^{n-1} v^t \ {}_tP_x $$

Illustration of cash value with a timeline as shown below

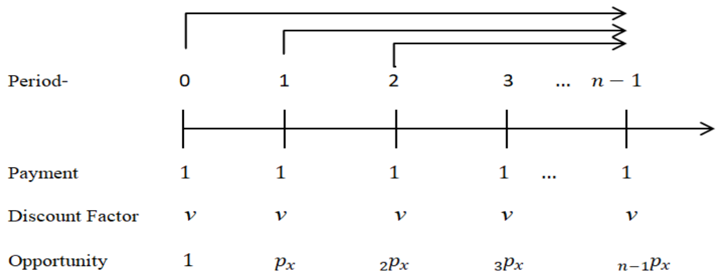

Figure 1. Early life annuity cash value timeline.

3. Result and Discussion

3.1. Life Futures Annuity Using the Cox-Ingersoll-Ross Interest Rate

For term n, the initial life annuity value is obtained using the CIR interest rate, namely

$$\ddot{a}_{x:\overline{n|}} = \sum_{t=0}^{n-1} \left(\prod_{s=1}^{t} \frac{1}{e^{-\beta s r_0} + \alpha(1-e^{-\beta s})} \right) \left(\frac{l_{x+t}}{l_x} \right) \tag{2}$$

Then by using equation (2), the initial value of a lifetime annuity is obtained for age x years with a coverage period of m years for m<n using the CIR interest rate, namely

$$\ddot{a}_{x:\overline{m|}} = \sum_{t=0}^{m-1} \left(\prod_{s=1}^{t} \frac{1}{e^{-\beta s r_0} + \alpha(1-e^{-\beta s})} \right) \left(\frac{l_{x+t}}{l_x} \right) \tag{3}$$

Based on equation (3), the initial life annuity value is obtained for the age (x+t) years with the coverage period (m-t) years, namely

$$\ddot{a}_{x+t:\overline{m-t|}} = \sum_{k=0}^{m-t-1} \left(\prod_{s=1}^{k} \frac{1}{e^{-\beta s r_0} + \alpha(1-e^{-\beta s})} \right) \left(\frac{l_{x+t+k}}{l_{x+t}} \right)$$

While the value of the initial life annuity with the coverage period of h years for $h < m < n$ is

$$\ddot{a}_{x:\overline{h|}} = \sum_{t=0}^{h-1} \left(\prod_{s=1}^{t} \frac{1}{e^{-\beta s r_0} + \alpha(1-e^{-\beta s})} \right) \left(\frac{l_{x+t}}{l_x} \right)$$

For insurance participants aged (x+t) years with a coverage period (h+t) years, it can be stated as

$$\ddot{a}_{x+t:\overline{h+t|}} = \sum_{k=0}^{h-t-1} \left(\prod_{s=1}^{k} \frac{1}{e^{-\beta s r_0} + \alpha(1-e^{-\beta s})} \right) \left(\frac{l_{x+t+k}}{l_{x+t}} \right)$$

3.2. Dwiguna Life Insurance Premium with CIR Interest Rate

According to Futami[6], term life insurance is insurance where the sum assured will be received if the insured dies during the contract period. Single premium term life insurance for individual status with policy contract for n years and sum assured of R unit of payment paid at the end of the policy year can be stated as follows:

$$\ddot{a}^{1}_{x:\overline{n|}} = R \sum_{t=0}^{n-1} v_t^{t+1} | q_x$$

A single premium term life insurance is obtained with a policy contract for n years and the sum insured are R units of payment using the CIR interest rate, namely

$$\ddot{a}^{1}_{x:\overline{n|}} = R\sum_{t=0}\left(\prod_{t=0}^{n-1}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\right)\left(\frac{l_{x+t}}{l_x}-\frac{l_{x+t+1}}{l_x}\right)$$

Single premium dual-purpose life insurance [3],[6] from participants aged x with a coverage period of n years which is denoted by $A_(xy:\overline{n|})$ expressed by

$$A_{x:\overline{n|}} = A^{1}_{x:n|} + A_{x:\frac{1}{n|}},$$

$$\ddot{a}_{x:\overline{n|}} = R\sum_{t=0}^{n-1} v^{t+1}\ {}_{t|}q_x + R(v^n\ {}_nP_{xy}),$$

Single premium dual-purpose life insurance from participants aged x with a coverage period of n years using the CIR interest rate, namely

$$A_{x:\overline{n|}} = R\sum_{t=0}^{n-1}\left(\prod_{s=1}^{t+1}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\right)\left(\frac{l_{x+t}}{l_x}-\frac{l_{x+t+1}}{l_x}\right)$$

$$+R\left(\prod_{s=1}^{n}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\right)\left(\frac{l_{x+n}}{l_x}\right)$$

Single premium dual-purpose life insurance for participants aged $(x+t)$ years and the coverage period for $(n-t)$ years is obtained

$$A_{x+t:\overline{n-1|}} = R\sum_{k=0}^{n-t-1}\left(\prod_{s=1}^{k+1}\frac{1}{e^{-\beta s}+\alpha(1-e^{-\beta s})}\right)\left(\frac{l_{x+t}}{l_x}-\frac{l_{x+t+1}}{l_x}\right)$$

$$+R\left(\prod_{s=1}^{n-t}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\right)\left(\frac{l_{x+t+n}}{l_x+t}\right)$$

the amount of the annual premium on dual-purpose life insurance for participants aged x years participating in insurance for n years can be expressed by

$$p_{x:\overline{n|}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}}$$

The annual premium for dual-purpose life insurance is obtained using the CIR interest rate, namely

$$P_{x:\overline{n|}} = \frac{R\sum_{t=0}^{n-1}\left(\prod_{s=1}^{t+1}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\right)\left(\frac{l_{x+t}}{l_x}-\frac{l_{x+t+1}}{l_x}\right)}{\sum_{t=0}^{n-1}\left(\prod_{s=1}^{t}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\right)\left(\frac{l_{x+t}}{l_x}\right)}$$

,

$$+\frac{R\Big(\prod_{s=1}^{n}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\big(\frac{l_{x+n}}{l_x}\big)}{\sum_{t=0}^{n-1}\Big(\prod_{s=1}^{n}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\big(\frac{l_{x+n}}{l_x}\big)}$$

The annual premium for participants aged x years participating in insurance for n years and the payment period for m years is,

$$P_{x:\overline{n}|}=\frac{R\sum_{t=0}^{n-1}\Big(\prod_{s=1}^{t+1}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t}}{l_x}-\frac{l_{x+t+1}}{l_x}\Big)}{\sum_{t=0}^{m-1}\Big(\prod_{s=1}^{t}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t}}{l_x}\Big)},$$

$$\frac{+R\Big(\prod_{s=1}^{n}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\big(\frac{l_{x+n}}{l_x}\big)}{\sum_{t=0}^{m-1}\Big(\prod_{s=1}^{t}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\big(\frac{l_{x+t}}{l_x}\big)}$$

3.3. Prospective Reserves and Zillmer Reserves at CIR Interest Rate

Prospective dual-purpose life insurance premium reserves for someone aged x years where t is the reserve calculation time, m is the premium payment period, and n is the coverage period with the sum assured paid at the end of the policy year where $t < m < n$ is,

$$ {}_{v}^{m}V_{x:\overline{n}|}=\begin{cases} A_{x+t:\overline{n-t}|}+_{m}P_{x:\overline{n}|}\ddot{a}_{x+t:\overline{m-t}|}, & t<m<n\\ A_{x+t:\overline{n-t}|}, & m\le t<n' \end{cases}$$

prospective reserves of dual-purpose life insurance are obtained for a period of $t < m < n4$, namely

$${}_{t}^{m}V_{x:\overline{n}|}=\Big(R\sum_{k=0}^{n-t-1}\Big(\prod_{s=1}^{k+1}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t+k}}{l_{x+t}}-\frac{l_{x+t+k+1}}{l_{x+t}}\Big)$$

$$=+R\Big(\prod_{s=1}^{n-t}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t+n_t}}{l_x+t}\Big)\Big)$$

$$+\Big(\Big(\frac{R\sum_{t=0}^{n-1}\Big(\prod_{s=1}^{t+1}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t}}{l_x}-\frac{l_{x+t+1}}{l_x}\Big)}{\sum_{t=0}^{m-1}\Big(\prod_{s=1}^{t}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t}}{l_x}\Big)}$$

$$\sum_{k=0}^{m-t-1}\Big(\prod_{s=1}^{k}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t+k}}{l_{x+t}}\Big)$$

Zillmer reserves are a type of modified premium reserves that are calculated using prospective reserves and a Zillmer rate of α. A. Zillmer is of the opinion that the loading which is the difference between the gross premium and the net premium in the 1st policy year is greater than the standard loading, so it is necessary to find a way to make the loading smaller than the standard loading. In this study, the value of α which is the Zillmer level is assumed to be 0.025.

Suppose ${}_mP_{x:\overline{n|}}$ is the annual premium of a person aged x years with an insurance period of n years and a premium payment period of m years, P_1 is the net premium in the policy year. 1, P_2 is the net premium in the $2nd$ to h policy years for $h < n$. P_1 and P_2 are modified premiums, where h is the Zillmer time and α is the Zillmer level so that $P_1 <_m P_{x:\overline{n|}} < P_2$, then

$$P_2 - P_1 = \alpha$$

$$P_2 = P_1 + \alpha$$

By paying premiums during the year, the premiums to be paid by insurance participants aged 20 years can be illustrated with a timeline as follows:

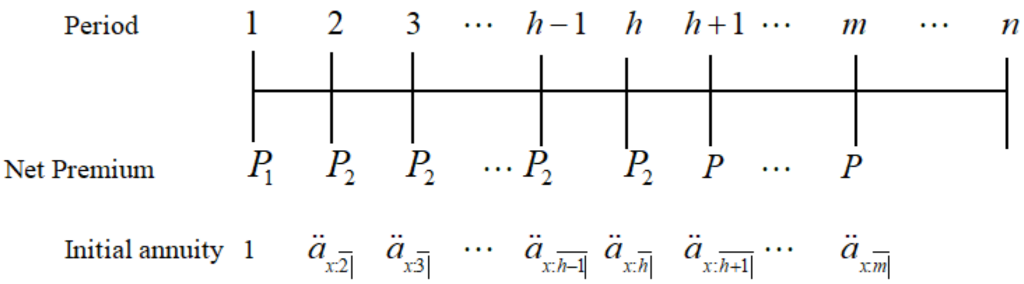

Figure 2. Modified premium timeline.

From the timeline in Figure 2, it can be seen that the modified premium cash value of the total premium income during the insurance period is fixed, the only thing that changes is the process of collecting net premiums so that the values of P_1 and P_2 can be determined as follows:

$${}_mP_{x:\overline{n|}}\ddot{a}_{x:\overline{m|}} = P_1 + P_2(\ddot{a}_{x:\overline{h|}} - 1) + P(\ddot{a}_{x:\overline{m|}} - \ddot{a}_{x:\overline{h|}}),$$

$$-P_2(\ddot{a}_{x:\overline{h|}} - 1) = P_1 + P\ddot{a}_{x:\overline{m|}} - P\ddot{a}_{x:\overline{h|}} -_m P_{x:\overline{n|}}\ddot{a}_{x:\overline{m|}}$$

$$-P_2 = \frac{P_1 + P\ddot{a}_{x:\overline{m|}} - P\ddot{a}_{x:\overline{h|}} -_m P_{x:\overline{n|}}\ddot{a}_{x:\overline{m|}}}{\ddot{a}_{x:\overline{h|}} - 1}$$

$$P_1 =_m P_{x:\overline{n|}} - \alpha + \frac{\alpha}{\ddot{a}_{x:\overline{h|}}}$$

$$P_2 = \frac{\alpha}{\ddot{a}_{x:\overline{h|}}} + {}_mP_{x:\overline{m|}}$$

Furthermore, the Zillmer reserve for someone aged x years with a Zillmer time of h years, a payment period of m years, and a coverage period of n for $(h < m < n)$, denoted by ${}^m_tV^{hz}_{x:\overline{n|}}$ for $1 \leq t \leq h$ is expressed as follows:

$${}^m_tV^{(hz)}_{x:\overline{n|}} = (A_{x+t:\overline{n-t|}} -_m P_{x:\overline{n|}}\ddot{a}_{x+t:\overline{m-t|}}) - \frac{\alpha}{\ddot{a}_{x:\overline{h|}}}\ddot{a}_{h+t:\overline{h-t|}}$$

$${}^m_tV^{(hz)}_{x:\overline{n|}} =^m_t V_{x:\overline{n|}} - \frac{\alpha}{\ddot{a}_{x:\overline{h|}}}\ddot{a}_{h+t:\overline{h-t|}}$$

Zillmer reserves from insurance participants aged x years with a coverage period of n years and a payment period of m years with a certain Zillmer α value and using the CIR interest rate, that is

$$
\begin{aligned}
{}^{m}_{t}V^{(hz)}_{x:\overline{n}|} = &\Bigg(\Bigg(\sum_{k=0}^{n-t-1}\Big(\prod_{s=1}^{k+1}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t+k}}{l_{x+t}}-\frac{l_{x+t+k+1}}{l_{x+t}}\Big)\\
&+R\Big(\prod_{s=1}^{n-t}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t+n_t}}{l_x+t}\Big)\Bigg)\\
&+\Bigg(\Bigg(\frac{R\sum_{t=0}^{n-1}\Big(\prod_{s=1}^{t+1}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t}}{l_x}-\frac{l_{x+t+1}}{l_x}\Big)}{\sum_{t=0}^{m-1}\Big(\prod_{s=1}^{t}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t}}{l_x}\Big)}\\
&+\sum_{k=0}^{m-t-1}\Big(\prod_{s=1}^{k}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t+k}}{l_{x+t}}\Big)\Bigg)\\
&\cdot\sum_{k=0}^{m-t-1}\Big(\prod_{s=1}^{k}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t+k}}{l_{x+t}}\Big)\Bigg)\Bigg)\\
&-\frac{\gamma}{\ddot{a}_{x:\overline{h}|}}\Bigg(\sum_{k=0}^{h-t-1}\Big(\prod_{s=1}^{k}\frac{1}{e^{-\beta s r_0}+\alpha(1-e^{-\beta s})}\Big)\Big(\frac{l_{x+t+k}}{l_{x+t}}\Big)\Bigg)\Bigg)\Bigg)
\end{aligned}
$$

Then, the estimated value is

$$\hat{\beta} = -\frac{1}{\delta j} ln\Big(\frac{q-\alpha o-\alpha p+n\alpha^2}{z-2\alpha p+n\alpha^2}\Big)$$

and,

$$\hat{\alpha} = \frac{oz-pq}{nz-nq+op-p^2}$$

Example.
A 35 year old male trader wants to take part in individual endowment life insurance with a term of 20 years, a premium payment period of 18 years, and a Zillmer term. If the sum assured received by the heirs is IDR 100,000,000.00 and the Indonesian interest rate from 2010 to 2019 then determine Zillmer Reserves for dual purpose life insurance using the Cox-Ingersoll-Ross interest rate and mortality table.
Is known:
$R = IDR100.000.000$,- $\quad x = 35$
$n = 20 \quad m = 18$
$z = 8$

In this example problem, to estimate the parameter values of the CIR interest rate model use data on Indonesian interest rates from 2010 to 2019. So that we get

$$\hat{\alpha} = \frac{(0,54250\cdot 0,03461)-(0,55125\cdot 0,033611)}{20(0,034612-0,033611)+0,55125(0,54250-0,55125)} = 0,059478665.$$

$$\hat{\beta} = -\frac{1}{\Delta 1} ln(\frac{0,03361 - (0,059480,54250) - (0,059480 \cdot 0,55125) + (20 \cdot 0,05948^2)}{0,03461 - (2 \cdot 0,05948 \cdot 0,55125) + (20 \cdot 0,059482)})$$

$\beta = 0,7995986060$

i Factor discount using the Cox-Ingersoll-Ross interest rate

$$v^t = \prod_{s=1}^{t} \frac{1}{e^{-0,7995986060s}(0,065) + 0,059478665(1 - e^{-0,7995986060s})}$$

ii The cash value of the initial life annuity has a term of 20 years using the obtained

$$\ddot{a}_{35:\overline{20|}} = \sum_{t=0}^{20-1} \Big(\prod_{s=1}^{t} \frac{1}{e^{-\beta s r_0} + \alpha(1 - e^{-\beta s})} \Big) \Big(\frac{l_{35+t}}{l_{35}} \Big) = 12,0135166421778$$

iii A single premium with a sum insured of IDR 100,000 000 is obtained

$$A_{35:\overline{20|}} = R \sum_{t=0}^{20-1} \Big(\prod_{s=1}^{t+1} \frac{1}{e^{-\beta s} + \alpha(1 - e^{-\beta s})} \Big) \Big(\frac{l_{35+t}}{l_{35}} - \frac{l_{35+t+1}}{l_{35}} \Big)$$

$$+R\Big(\prod_{s=1}^{20} \frac{1}{e^{-\beta s r_0} + \alpha(1 - e^{-\beta s})} \Big) \Big(\frac{l_{35+n}}{l_{35}} \Big)$$

iv The cash value of the initial annuity with a term of 18 years for which payment is obtained

$$\ddot{a}_{35:\overline{18|}} = \sum_{t=0}^{18-1} \Big(\prod_{s=1}^{t} \frac{1}{e^{-\beta s r_0} + \alpha(1 - e^{-\beta s})} \Big) \Big(\frac{l_{35+t}}{l_{35}} \Big) = 11,3668312315629$$

v The annual premium for dual-purpose life insurance with a coverage period of n years and a payment period of m years is

$${}_{18}P_{35:\overline{20|}} = \frac{A_{35:\overline{20|}}}{\ddot{a}_{35:\overline{20|}}} = IDR2.844.984,568$$

vi Initial annuity cash value with coverage period $(m-1)$ years and single premium with coverage period $(n-1)$ years

$$\ddot{a}_{35+1:\overline{120-1|}} = \sum_{k=0}^{18-1-1} \Big(\prod_{s=1}^{k} \frac{1}{e^{-\beta s r_0} + \alpha(1 - e^{-\beta s})} \Big) \Big(\frac{l_{35+t}}{l_{35}} \Big) = 10,9957465187483$$

vii The single premium for $(n-1)$ years earned

$$A_{35+1:\overline{20-1|}} = R \sum_{t=0}^{20-1-1} \Big(\prod_{s=1}^{k+1} \frac{1}{e^{-\beta s} + \alpha(1 - e^{-\beta s})} \Big) \Big(\frac{l_{35+1+k}}{l_{35+1}} - \frac{l_{35+1+k+1}}{l_{35+1}} \Big)$$

$$+R\Big(\prod_{s=1}^{20} \frac{1}{e^{-\beta s r_0} + \alpha(1 - e^{-\beta s})} \Big) \Big(\frac{l_{35+1+20-1}}{l_{35+1}} \Big)$$

$$A_{35+1:\overline{20-1|}} = IDR34.202.893,9315831$$

viii The initial life annuity of the insured period h years using equation (2.33) is obtained

$$A_{35:\overline{8|}} = \sum_{t=0}^{8-1} \left(\prod_{s=1}^{t} \frac{1}{e^{-\beta s} + \alpha(1 - e^{-\beta s})} \right) \left(\frac{l_{35+5}}{l_x} \right) = 6,54872586636174$$

ix For the period of coverage $(h-1)$ obtained

$$A_{35-1:\overline{8-1|}} = \sum_{k=0}^{8-1-1} \left(\prod_{s=1}^{k} \frac{1}{e^{-\beta s} + \alpha(1 - e^{-\beta s})} \right) \left(\frac{l_{35+1+k}}{l_{35+1}} \right) = 5,88818035968421$$

Zillmer reserves using the CIR interest rate for the first year are

$${}^{18}_{1}V^{(8z)}_{\overline{35:20|}} = (2920164,769) - \frac{0.025}{6,54872586636174} 5,88818035968421$$

$${}^{18}_{1}V^{(8z)}_{\overline{35:20|}} = IDR2.920.164,747$$

So, the amount of reserves available to the insurance company at the end of the first year of the contract for a 35-year-old male trader is IDR 2,920,164,747. The complete calculation of Zillmer reserves for endowment life insurance with an interest rate of CIR from a 35-year-old man with Zillmer time of $h = 8$ years in year t, for the two cases above is presented in Table 1

t	Single Premium (IDR) $A_{35+t:\overline{20-t|}}$	Zillmer Reserves (IDR) ${}^{18}_{t}V^{(8z)}{}_{25:\overline{20|}}$
0	32.338.459,4437441	0,025
1	34.202.893,9315831	2.920.164,747
2	36.175.055,6751705	6.009.476,075
3	38.260.549,8079212	9.276.864,061
4	40.466.077,2136807	12.732.924,47
5	42.797.139,7075944	16.386.557,96

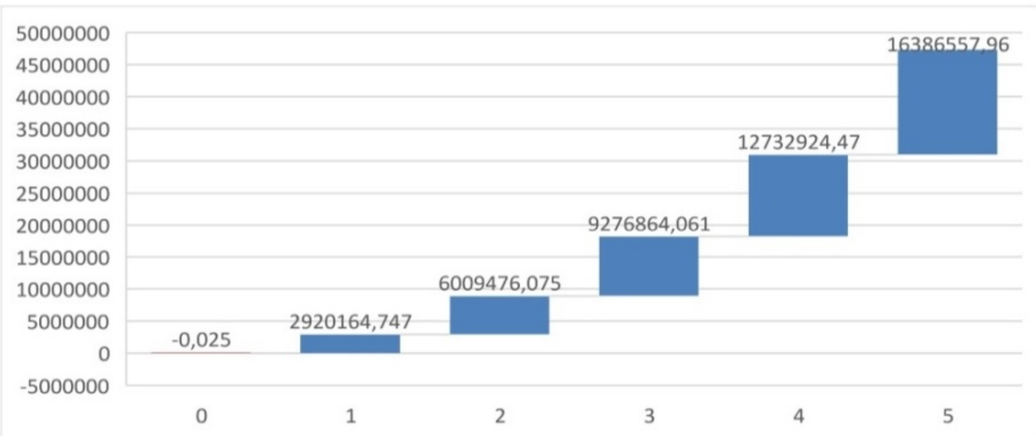

Figure 3. Zillmer Reserve Chart.

4. Conclusion

This research has discussed Zillmer reserves in dwiguna life insurance with the Cox-Ingersoll-Ross (CIR) interest rate. Based on the results of the analysis it can be concluded that the Zillmer reserve is an alternative method that can be used to determine the reserve derived from the net premium with a value of α which is the Zillmer rate. In this case, a modification of prospective reserves is made using the CIR interest rate. It is expressed in the form of a discount vector, so an increasing reserve is obtained. This research is very useful for insurance companies in predicting the reserves that must be owned by the insurance company.

Suggestions for future research, determining Zillmer reserves in Dwiguna Life Insurance, apart from using the Cox-Ingersoll-Ross (CIR) interest rate, can be combined with the Zillmer population model, referring to Pflaumer (2011). DOI:10.1007/ 978-3-319-03122-4_22.

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References

[1] L. J. Bain and M. Engelhardt, “Introduction to probability and mathematical statistics”, Second Ed., Duxbury, California, 1992.

[2] N.L. Bowers, H. U. Geerber, J. C. Hikcman, D.A. Jones and C.J. Nessbitt, “Actuarial mathematics,” Second Ed., The Society Of Actuaries, Schaumburg, 1997.

[3] D.C.M. Dickson, M. R. Hardy and H. R. Waters, “Actuarial mathematics for life contingent risk,” Cambridge University Press, Cambridge, 2009.

[4] J. C. Cox, J. E. Ingersoll, S. A. Ross, “A theory of the term structure of interest”, Econometrica,53 (1985).385-407, 1985. http://www.fin.ntu.edu.tw/ tzeng/course/CIR(1985)-Eca.pdf

[5] M. B. Finan, “A reading of life contingency model: A preparation for exam MLC/3L,” Arkansas Teach University, Arkansas, 2011.

[6] T. Futami, “Matematika asuransi jiwa, bagian I, terjemahan. dari seimei hoken sugaku, gekan (92 revision),” by G. Herliyanto, Oriental Life Insurane Cultural Development Center, Tokyo, 1993.

[7] T. Futami, “Matematika asuransi jiwa bagian II,” terjemahan. dari seimei hoken sugaku, gekan (92 revision),” oleh G. Herliyanto, Oriental Life Insurane Cultural Development Center, Tokyo, 1994.

[8] Hasriati and T. P. Nababan, “Private premium of endowment last survivor and joint life insurance with pareto distribution,” International Journal of Statistical Distribution and Applications, 4 No. 5 (2019): 76-81, 2019. doi: 10.11648/j.ijsd.20190504.11.

[9] Hasriati, T. P. Nababan, I. Hasbiyati, Sukono, and A.T. Bon, “Reserve of life insuren prospective dwiguna joint life and last survivor with Gompertz law,”, Proceedings of the 2nd African International Conference on Industrial Engineering and Operations Management Harare, Zimbabwe December 7-10, 2020. http://www.ieomsociety.org/harare2020/papers/434.pdf

[10] S. G. Kellison, “The theory of interest,” Richard D. Irwin Inc, Homewood, 1991.

[11] Robert Musian, “Mathematics of interest rate, insurance, social security, and pensions”, Prentice-Hall, London, 2003.

[12] Sukono, K. Joebaedi, N. Dora, E.S. Hasbullah, D. Susanti, Hasriati, and A.T. Bon, “Risk surplus analysis in credit life insurance using bayesian method,” International Conference on Industrial Engineering and Operations Management Detroit, Michigan, USA. 10-14, 2020. http://www.ieomsociety.org/detroit2020/papers/506.pdf